COMPARTIVE STUDY FOR DIFFERENT TRANSFORMATION MODELS APPLIED ON GEODETIC COORDINATE SYSTEMS

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تتطلب التنمية الاقتصادية لاى دولة انشاء شبكات حديثة من نقط التحكم الارضية وذلك لاغراض التنمية و انشاء مجتمعات عمرانية جديدة وكذلك عمل مشاريع دولية بين البلاد المختلفة وذلك كلة يتم من خلال خرائط مساحية.
وهذة الخرائط تعتمد على نقط التحكم الارضية ، ومن المعروف ان ايجاد احداثيات هذة النقاط هو واحد من اهم أهداف علم الجيوديسيا ولكى نحدد موضع نقطة على سطح الارض يتم ذلك ابتداءا بانشاء الشبكات التقليدية مرصودة الزاويا والاطوال، ثم يليها مرحلة الحسابات والتى تتم على سطح مرجع مناسب والذى من خصائصة ان يكون اقرب ما يكون لسطح البحر او الجيويد، وهذا الاسلوب التقليدى يحتاج الى جهد ووقت كبير.
ومع التقدم المتزايد فى التكنولوجيا واستخدام تقنيات عالية الدقة لحساب الاحداثيات تم استخدام GPS كنظام بديل عن الطرق التقليدية حيث انة يعطى دقة عالية مع توفير الوقت والجهد،ولكن يعطى هذا النظام احداثياتة على سطح الاسناد العالمى WGS84
وبالتالى كان لابد من دراسة عملية التحويلات من نظام احداثيات الى نظام اخر باستخدام النماذج الرياضية المختلفة واختيار انسب نموذج يعطى اقل قيم فروقات (residuals) عند نقط المراجعة(Check points).
ليس الغرض من البحث عمل او الحصول على معاملات للتحويل فى منطقة الدراسة، ولكن الغرض الرئيسى هو مقارنة النماذج الرياضية المختلفة للتحويل وذلك لتحديد ايهم احسن استخداما في هذا المجال.
فى هذا البحث تم استخدام العديد من النماذج الرياضية بطرق مختلفة نتج عنها العديد من الحلول التي استخدمت في المقارنة لاختيار احسنها. وضعت نتائج الحلول واحصائاتها فى جداول، وتم تمثيل هذة النتائج فى أشكال (هيستوجرام). أخيرا تم اختيار أفضل نموذج للتحويل فى بعدين وكذلك أفضل نموذج للتحويل فى الثلاثة أبعاد.

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ABSTRACT

Positioning is one of the main tasks of geodesy. In order to define the position of any point on the earth's surface, it is necessary to determine its position referred to a (2D) or (3D) coordinate system. The geodetic-positioning network in Egypt has been traditionally established at the first decades of the last century. Helmert-1906 ellipsoid is adopted and initial point (F1) was chosen, so the Egyptian datum is defined at that time.

Nowadays GPS plays an important role in establishing the global and national positioning networks. GPS is also used in strengthening the old (traditional) national networks. GPS is precise technique compared with the traditional ones. For example. the original Network I in Egypt has a precision around 1 : 100,000. While the GPS network (HARN), made by Egyptian Surveying Authority (ESA) in 1997, has a precision of 1 : 10,000,000 which is 100 times more accurate.

The traditional network of Egypt is defined on Helmert 1906 which is a local datum. GPS networks are defined on the Geo-centric datum WGS-84. So, for many obvious reasons, coordinate transformation between the two systems is required.

Obtaining a definite-precise set of tramsformation parameters is a goal of many researchers. The transformation process depends on some factors among them is the used mathematical model. Obtaining transformation set of parameters is not the target of this research. The subject of this research is studying different transformation models and investigating the best among them.

Several transformation models are applied and investigated using different solutions. The results are tabulated and represented in histograms for the sake of comparisons. The best model is chosen in the case of 2D and 3D transformations.

1-Introduction

In Egypt, the first order geodetic triangulation network started at 1907 used Helmert 1906 as a reference datum. In 1977 the American Defense Mapping Agency (DMA) established the first satellite stations known by DOPPLER system which used a geocentric datum (WGS-72), and in 1990 the Finnmap project established 300 control points depending on the world geodetic datum (WGS–84). In 1997, ESA established the High Accurate Reference Network which is known as (HARN). Other GPS networks are initiated in Egypt and they could be connected to the Egyptain network to strengthen the geodetic field.

So. the transformation process between two different datums, using transformation parameters. is required. In this research, the goal is not to obtain new transformation parameters between the Local Egyptian Datum (LED) and the global (WGS-84) datums. The goal of this research is to investigate the best model which can be used in the transformation process.

2- Previous Works

[Bekheet, A. 1993] applied two transformation models namely Bursa model and first order polynomial in two dimensions. He applied the two models to find relationships among the three datums LED. WGS-72, WGS-84. He used 8 common points in the solution and there were no check points to assess the results. The residuals at the used solution points showed that the used polynomial gave better results than Bursa model.

[Abd-Elmotaal, H. 1994] presented the comparison of polynomial and similarity transformation based datum shifts for Egypt. He used 8 common points from first order geodetic stations known in both WGS-84 and old Egyptian datum as the solution points. These points are located in the Egyptian eastern desert, and thier WGS84 coordinates have been taken from the results of Finnmap project, 1989. He also used geoidal undulations computed by Finnmap, and there were no check points. The results showed that the used second order polynomial is better than the similarity transformation, Busra model.

[Fayad, A. T. 1996] studied three transformation models (Moldonsky. Bursa, and ten parameters). He used 8 common points and determined the transformation parameters for each model. Also he determined the residuals at the solution points. and no check points are used. The ten parameters model was sligtly better than the similarity models.

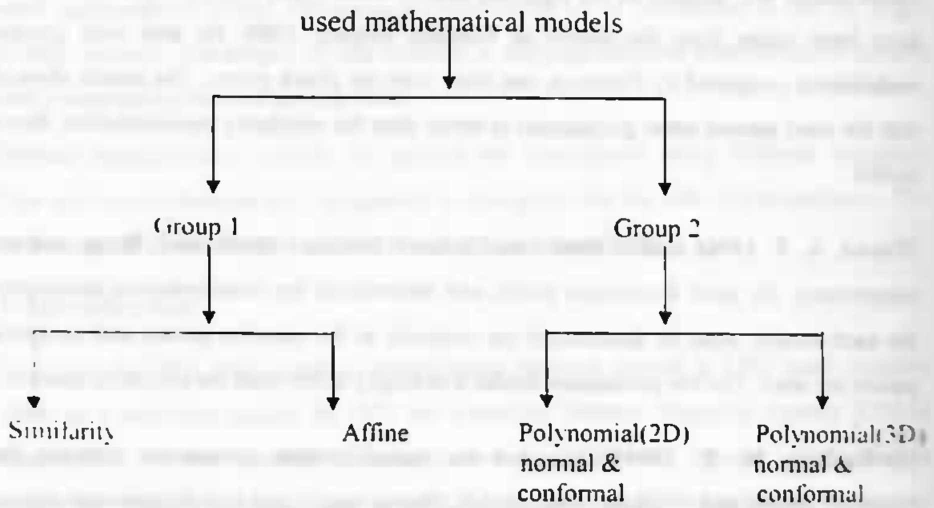
[El-Tokhey, M. E. 1999] computed the transformation parameters between the Egyptian datum and WGS-84. Two models (Bursa model and two dimensional-second order surface polynomial) are employed. Fifteen common points are used. They are first order points taken from the final adjusted coordinates of [Awad, M. E. 1997]. They are also defined on WGS-84 through the High Accuracy Referance Network (HARN).

The derived transformation parameters have been checked at other 16 stations from the Egyptian Aviation Authority (EAA). The results showed that the used polynomial is better than Bursa model.

[Gomaa, M. and D. S. Alnaggar, 2000] presented the optimum geodetic datum transformation techniques for GPS surveys in Egypt using (similarity models Bursa and Molodensky) and (two dimensional-multiple regression surface polynomials). The available coordinates were19 first order stations known in both WGS84 and old Egyptian datum. Fifteen points were used as solution points. The GPS coordinates are taken from (HARN) and the remaining stations have been observed by the Survey Research Institute (SRI) as part of the Egyptian National Standardization Gravity Network (ENSGN97). Four stations have been considered as check points. The results showed that multiple regression surface polynomial is better than Bursa and Molodensky.

3- Transformation Models

In this section, the used transformation models will be classified and explained. The models are firstly classified into two main groups as follows:



3-1 Group (1)

3-1-1 Similarity Transformation in Three Dimensions

Similarity means that the scale factor is the same in all directions. When transforming from one spatial coordinate system to another we need seven parameters (linear conformal) three parameters for translation, one scale factor and three rotation parameters. Bursa and Molodensky models will be used in this research and are explained as follows [**Thomson, 1976**];

i) Bursa Model

This model is considered as the most common model in determining transformation parameters between any two different 3D coordinate systems. The mathematical form of this model is given as:

$$X_o + (1+K)\, R\, X_p - x_p = 0 \qquad (1)$$

Where

- X_o : is the translation vector between two origins of coordinate system
- R : Rotation matrix = R1 (wx) R2 (wy) R3 (wz)
- 1+K : Scale factor
- x_p : Position vector of terrain point "P" in one system
- X_p : Position vector of terrain point "P" in the other system

To solve for the seven unknown parameters, at least three common points defined in both systems should be available. To apply the least squares adjustment we need 3 common points at least. For more details about least squares adjustment, reference is made to [**Nassar, 1981**].

ii) Molodensky Model

This model describes the relation between any two different 3-D coordinate systems by seven parameters. It is described mathematically as:

$$Xo + Xi + (1 + k)\, R\, \Delta Xip - xp = 0 \qquad (2)$$

The equation is similar to Bursa model except the new vector (Xi) which is the position vector of the initial point (i). Also the axes of the two systems are parallel so that the rotation and the scale are only applied on the vector ΔXip between any point and the initial point.

3-1-2 Affine Transformation in 3D (Ten Parameters Model)

Affine Transformation considers different scale factors along the different axes. The coordinates in three dimensional terrestrial systems are derived from a horizontal triangulation and/or trilateration networks and from leveling networks (plus ellipsoidal heights if available). The scale of these two different networks, horizontal and vertical, is different. In addition a systematic distortion should be accounted for in both horizontal and vertical networks. The ten parameters transformation model accounts for these two main sources of errors [Fayed, A. T. 1996], the ten parameters are;

x_o, y_o, z_o	:	The shift components between the terrestrial and the satellite coordinate systems.
w_x, w_y, w_z	:	The rotation elements of the local geodetic system, at initial point of the terrestrial network, with respect to the terrestrial system.
α	:	The horizontal direction of the maximum scale distortion.
k_1, k_2	:	The scale factors which model the distortion in the horizontal plane of the terrestrial network.
K_3	:	The scale factor which models the distortion in the vertical direction of the terrestrial network

The basic idea is considering the coordinates of satellite geocentric coordinate system (X, Y, Z) to be transformed into terrestrial coordinate system (x, y, z).

The final transformed point vector to the terrestrial coordinate system is indicated by x_p, while Xo is the shift component vector between the geocentric and the terrestrial systems. Neglecting the second order terms, the linearized form of the transformation equation takes the following form:

$$x_p = X_o + X_p + M^T (dR + dS) M \Delta X_{ip} \quad (3)$$

where:

$$M = \begin{bmatrix} -\sin\varphi_i\cos\lambda_i & -\sin\varphi_i\sin\lambda_i & \cos\varphi_i \\ -\sin\lambda_i & \cos\lambda_i & 0 \\ \cos\varphi_i\cos\lambda_i & \cos\varphi_i\sin\lambda_i & \sin\varphi_i \end{bmatrix} \quad (4)$$

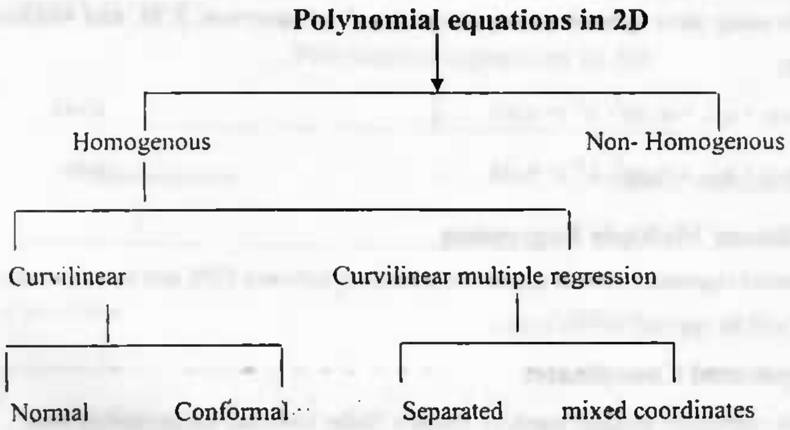
The least squares adjustment of observations and independent parameters can be used to obtain the ten unknown transformation parameters.

3-2 Group 2: Coordinate Transformation Using Polynomials

Polynomial equations are used to represent different shapes of surfaces depending on the degree of the polynomial. The first-degree polynomial trend surface is a plane, without any curvature. The other degrees of polynomial shows different curved surfaces depending on the number of the used coefficients. The surface is normally used to best fit the data points with minimum differences. The produced surface is tested by the residuals at the data points and at other check points. The confidence on the polynomial depends on the number of the used data points, their quality in both systems, and their distribution in the considered area.

3-2-1 Coordinate Transformation Using 2D Polynomials

The polynomials used in two dimension transformation are classified in the following diagram:



Homogenous means relating two coordinate systems of the same type to each other, i.e. φ, λ to φ', λ'. Non-Homogenous means relating two different types of coordinate system to each other, i.e. φ, λ to X, Y.

3-2-1-1 Homogenous Polynomial in Two Dimensions

(i - a) Curvilinear Normal Case in 2D

This model computes corrections to φ and λ to change them into corresponding values in the other system. The model takes the form:

$$\Delta\varphi = a_0 + a_1\varphi + a_2\lambda + a_3\varphi^2 + a_4\varphi\lambda + a_5\lambda^2 + \ldots.. \quad (5\text{-a})$$

$$\Delta\lambda = b_0 + b_1\varphi + b_2\lambda + b_3\varphi^2 + b_4\varphi\lambda + b_5\lambda^2 + \ldots.. \quad (5\text{-b})$$

The polynomials can obviously be extended to higher powers in φ and λ, where (in the case of transforming WGS84 coordinates to Helmert 1906 coordinates);

$\Delta\varphi = \varphi_{(WGS84)} - \varphi_{(1ED)}$,

φ , λ geodetic coordinates in WGS84.

a_i and b_i , i = 0 to n are the unknown coefficients of the polynomial.

(i – b) Curvilinear Conformal Case in 2D

The conformal property preserves the angles between intersecting lines after the transformation. The conformality could be applied through the two following polynomials using same symboles as equations 5-a, 5-b [**Anserson, J. M. and Mikhail, E. M. 1998**]:

$$\Delta\varphi = a_0 + a_1\varphi + a_2\lambda + a_3(\varphi^2 - \lambda^2) + a_4\varphi\lambda \ \ldots.. \quad (6\text{-a})$$

$$\Delta\lambda = b_0 + b_1\varphi + b_2\lambda + b_3(\varphi^2 - \lambda^2) + b_4\varphi\lambda \ \ldots.. \quad (6\text{-b})$$

(ii) Curvilinear Multiple Regression

The polynomial regression model applied to transform between GPS and local systems. The model will be applied in two styles:

(ii - a) Separated Coordinates

The multiple regression process starts by fitting a linear function, the procedure then sequentially adds one variable at a time to the equation. Finally, the polynomial takes the form [Gomaa, M. and D. S. Alnaggar, 2000]:

$$\Delta\varphi = a_0 + a_1\varphi + a_2\lambda + a_3\varphi^2 + a_4\varphi^3 + a_5\lambda^2 + a_6\lambda^3 + a_7\varphi^4 + a_8\lambda^4 \qquad (7\text{-a})$$

$$\Delta\lambda = b_0 + b_1\lambda + b_2\varphi + b_3\lambda^2 + b_4\lambda^3 + b_5\varphi^2 + b_6\varphi^3 + b_7\lambda^4 + b_8\varphi^4 \qquad (7\text{-b})$$

(ii - b) Mixed Coordinates

In this model we relate the coordinates to the centroid point in the area, and add one variable to the equation after first order case as;

$$\Delta\varphi = a_0 + a_1(\varphi - \varphi_m) + a_2(\lambda - \lambda_m) + a_3(\varphi - \varphi_m)^2 + a_4(\varphi - \varphi_m)(\lambda - \lambda_m) + a_5(\lambda - \lambda_m)^2 \quad \ldots(8\text{-a})$$

$$\Delta\lambda = b_0 + b_1(\varphi - \varphi_m) + b_2(\lambda - \lambda_m) + b_3(\varphi - \varphi_m)^2 + b_4(\varphi - \varphi_m)(\lambda - \lambda_m) + b_5(\lambda - \lambda_m)^2 \quad \ldots \ (8\text{-b})$$

where φm and λm are the geodetic coordinates for centroid point at (WGS84)

3-2-1-2 Non-Homogenous Case in 2D

This model computes corrections to the rectangular coordinates X.Y.Z from the horizontal coordinates φ . λ of the other system. The polynomials take the form:

$$\Delta X = a_0 + a_1\varphi + a_2\lambda + a_3\varphi^2 + a_4\varphi\lambda + a_5\lambda^2 + \ldots \qquad (9\text{-a})$$

$$\Delta Y = b_0 + b_1\varphi + b_2\lambda + b_3\varphi^2 + b_4\varphi\lambda + b_5\lambda^2 + \ldots \qquad (9\text{-b})$$

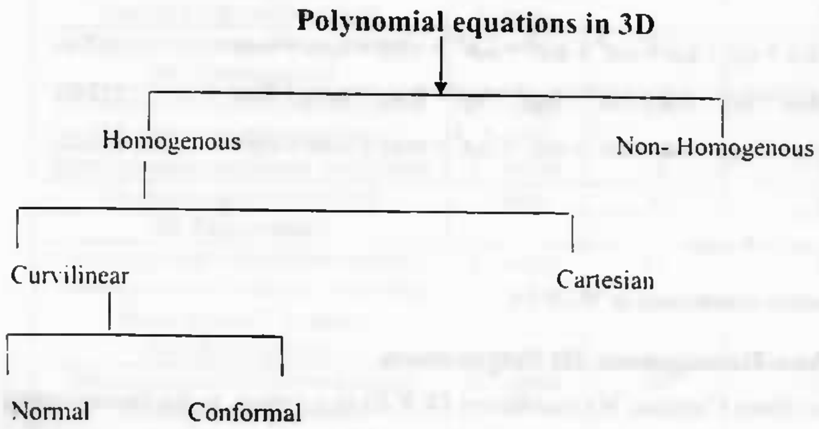
$$\Delta Z = c_0 + c_1\varphi + c_2\lambda + c_3\varphi^2 + c_4\varphi\lambda + c_5\lambda^2 + \ldots \qquad (9\text{-c})$$

where :

$\Delta X = X_{(WGS84)} - X_{(LLD)}$

3-2-2 Coordinate Transformation Using 3D Polynomials

Coordinate transformations using polynomials in three dimensions are employed in this research as follows:



3-2-2-1 Homogenous Polynomials in 3D

(i–a) Curvilinear Normal Case in 3D

This model relates the changes in (φ, λ, h) in a system function of (φ,λ, h) in the second system through normal polynomials as follows:

$\Delta\varphi = a_0 + a_1\varphi + a_2\lambda + a_3h + a_4\varphi^2 + a_5\lambda^2 + a_6h^2 + a_7\varphi\lambda + a_8\varphi h + a_9\lambda h$ (10-a)

$\Delta\lambda = b_0 + b_1\varphi + b_2\lambda + b_3h + b_4\varphi^2 + b_5\lambda^2 + b_6h^2 + b_7\varphi\lambda + b_8\varphi h + b_9\lambda h$ (10-b)

$\Delta h = c_0 + c_1\varphi + c_2\lambda + c_3h + c_4\varphi^2 + c_5\lambda^2 + c_6h^2 + c_7\varphi\lambda + c_8\varphi h + c_9\lambda h$ (10-c)

where :

φ . λ. h geodetic coordinates in WGS-84

(i – b) Curvilinear Conformal 3D Case

This model is exactly like the above model except it uses conformal polynomials as follows:

$\Delta\varphi = a_0 + a_1\varphi + a_2\lambda + a_3h + a_4(\varphi^2 - \lambda^2 - h^2) + a_5\varphi\lambda + a_6\varphi h$ ..... (11-a)

$\Delta\lambda = b_0 + b_1\varphi + b_2\lambda + b_3h + b_4(-\varphi^2 + \lambda^2 - h^2) + b_5\varphi\lambda + b_6\lambda h$ ..... (11-b)

$\Delta h = c_0 + c_1\varphi + c_2\lambda + c_3h + c_4(-\varphi^2 - \lambda^2 + h^2) + c_5\varphi h + c_6\lambda h$ ..... (11-c)

(ii) Cartesian 3D Polynomials

This model relates two Cartesian coordinate systems (X,Y,Z) and (x, y, z) to each other as follows:

$\Delta X = a_0 + a_1x + a_2y + a_3z + a_4x^2 + a_5y^2 + a_6z^2 + a_7xy + a_8xz + a_9yz + \ldots$ (12-a)

$\Delta Y = b_0 + b_1x + b_2y + b_3z + b_4x^2 + b_5y^2 + b_6z^2 + b_7xy + b_8xy + b_9xz + \ldots$ (12-b)

$\Delta Z = c_0 + c_1x + c_2y + c_3z + c_4x^2 + c_5y^2 + c_6z^2 + c_7xy + c_8xz + c_9yz + \ldots$ (12-c)

where :

$\Delta X = x_{(WGS84)} - X_{(LED)}$

x, y, z geodetic coordinates in WGS-84

3-2-2-2 Non-Homogenous 3D Polynomials

The model relates Cartesian 3D coordinates (X,Y,Z) in a system, to the corresponding curvilinear (φ,λ,h) coordinates in another system as follows:

$\Delta X = a_0 + a_1\varphi + a_2\lambda + a_3h + a_4\varphi^2 + a_5\lambda^2 + a_6h^2 + a_7\varphi\lambda + a_8\varphi h + a_9\lambda h$ ... (13-a)

$\Delta Y = b_0 + b_1\varphi + b_2\lambda + b_3h + b_4\varphi^2 + b_5\lambda^2 + b_6h^2 + b_7\varphi\lambda + b_8\varphi h + b_9\lambda h$ ... (13-b)

$$\Delta Z = c_0 + c_1\varphi + c_2\lambda + c_3h + c_4\varphi^2 + c_5\lambda^2 + c_6h^2 + c_7\varphi\lambda + c_8\varphi h + c_9\lambda h \quad \ldots \quad (13\text{-c})$$

where $\Delta X = x_{(WGS84)} - X_{(LED)}$

4- The Data Used in the Computations

4-1 Number of Known Points Needed in Computations

To determine the relation between any two different coordinate systems, common points are needed. The minimum number of these points depends on the used model and its degree. If the available points exceed the minimum, least squares adjustment can be used to obtain the best solution. So, the following are the unknown parameters and the needed minimum data points for each model:

Table (1) Number of unknown parameters and required data points.

Model	Degree	No of. Unknowns	No of minimum points
Bursa and Molodensky		7	3
Ten parameters		10	4
2D Polynomials			
Homogenous curvilinear normal Homogenous curvilinear conformal Non-homogenous	1^{st} -order	6	3
Homogenous curvilinear normal Non-homogenous Multiple regression mixed	2^{nd} -order 2^{nd} -order 6-terms	12	6
Homogenous curvilinear normal Non-homogenous	3^{rd} -order	20	10
Homogenous curvilinear conformal Multiple regression mixed	2^{nd} -order 5-term	10	5
Homogenous curvilinear conformal	3^{rd} -order	14	7
Multiple regression mixed	4-term	8	4
3D Polynomials			
Homogenous curvilinear normal Homogenous curvilinear conformal Homogenous Cartesian Non-homogenous	1^{st} -order	12	4
Homogenous curvilinear normal Homogenous Cartesian Non-homogenous	2^{nd} -order	30	10
Homogenous curvilinear conformal	2^{nd} -order	21	7

4-2 Data Sources

The available data used are obtained from three sources. The first is the Egyptian traditional triangulation networks which are related to Helmert-1906. The altitudes of the stations in ESA are modified into ellipsoidal heights through the geoid model (SRI2000B). The second set is taken from different (GPS) networks (HARN and Finnmap) after unifying them **[Saad, A. 1998]**, they are related to WGS-84. The precision of HARN vectors is 10,000,000 **[ESA Report, 1997]**. The total number of common points are 28 points but as we mentioned before, the data have different sources, with different accuracies. Therefore, a filtering scheme is followed to obtain consistent data set for the proposed computations. The filtering scheme is done on two stages;

First stage; using all the available common points, the resultant difference vectors at all points are computed as follows:

$$R = \sqrt{(X_{WGS84} - X_{LED})^2 + (Y_{WGS84} - Y_{LED})^2 + (Z_{WGS84} - Z_{LED})^2}$$

Mean and standard deviations (σ) are computed for the resultant residual vectors. The station with residual greater than 3σ is rejected. After applying this filtering, 16 stations are used. Second stage; using the above mentioned 16 points, adjacent triangles are traced. At every triangle, the differences in coordinates between the two systems (ΔX, ΔY, ΔZ) were calculated at the three vertices. The station with odd values was rejected. Thirteen stations are accepted after applying this filter. The final common points are 13 points, covering the eastern desert of Egypt, as illustrated in Figure (1).

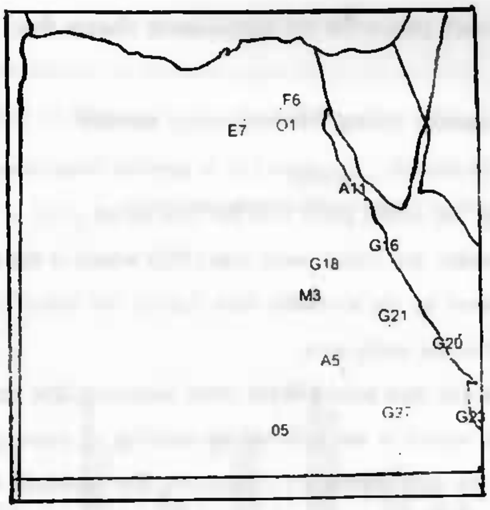


Figure (1) Configuration of the common points used in the study

5- Applications and the Results

Solutions based on the above mentioned models are done. The available 13 common points are divided into two groups. The first is used in computing the transformation parameters (data or solution points); residuals are computed at these stations. The second group is used as check points and the residuals are computed also at these check points.

Recalling that, obtaining transformation parameters is not our concern here, so they will not be represented. In [Abd-Elhay, 2004], detailed results are shown; statistics of the residuals at the data points and at the check points are tabulated and presented in histograms for the sake of comparison. In every solution and at every point, the residuals of the coordinates and their resultants are shown in one column. The results are analyzed and the best model for coordinate transformation is chosen. In this research, only summary of results are presented.

5-1 Group (1)

a- Simllarity Transformation using Bursa Model

The mathematical model of Bursa, equations (1), is applied. Then, the residuals at the solution points and their statistics are computed. The residuals at 4 check points are also computed. The residuals at check points did not significantly change from the residuals at the data points.

b- Similarity transformation using Molodensky model

The mathematical model of Moldensky, Equation (3), is applied three times using three different initial points. Firstly, the initial point was the real initial point of the geodetic network of Egypt (O1). Secondly, the initial point was (M3) which is the nearest point to the center of the area covered by the available data. Lastly, the initial point was the imaginary (calculated), center of the study area.

The residuals are computed at the data points in the three solutions. The three solutions have identical results, i.e the model is not affected by shifting its centeral point. The statistics of the residuals at the data points are computed. The residuals at the check points are computed from the three cases and they were also identical. Molodensky again is similar to Bursa model where it behaves the same way towards both data and check points.

c- Affine Transformation Using Ten Parameters Model

The mathematical model of the ten parameters model, Equation (4), is applied three times using three different initial points, similarly as in Molodensky case. The transformation parameters are computed in every case. The residuals at the data points are also computed and they are identical from the three cases, their statistics are also computed. The residuals at the check points are computed and they are also identical from the three solutions. The ten parameters model reacts the same way towards data and check points

d- The Best Solution for Group (1)

The residuals at the data points from the three models, Bursa, Molodensky, and Ten parameters are collected in order to choose the best model among them. The statistics of their resultant residuals are shown as follows;

Table(2) Statistics of resultant residuals of data points using three models in group 1

Statistical data	Bursa Resultant m	Molodensky Resultant m	Ten parameters Resultant m
Min	0.26	0.26	0.22
Max	1.85	1.85	1.81
Mean	0.75	0.76	0.67
ST.dev.	0.51	0.51	0.49

The resultant residuals at the data points from the three models in group 1 are collected and represented as follows:

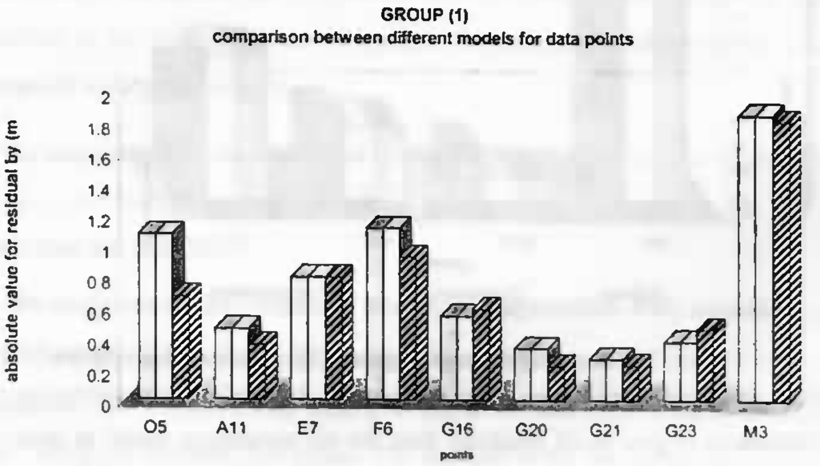


Figure (2) Resultant residuals of group (1) models at data points

From Table (2) and Figure (2) above, it can be seen that the affine transformation (10-parameters) model gives small residuals than the other two models which are already identical. So, the best model in group1, at solution points is the (Ten parameters) model. The resultant residuals at the check points from the three solutions are shown in Table (3) and represented in Figure (3) in order to compare the models at those points.

Table (3) The resultant residuals at check points using models of group (1)

POINT	Resultant Residuals m		
	Bursa	Molodensky	10- parameters
A5	1.81	1 81	1 95
G18	0.18	0.18	0.38
G27	0 55	0.55	0 63
O1	1.10	1.10	0.92

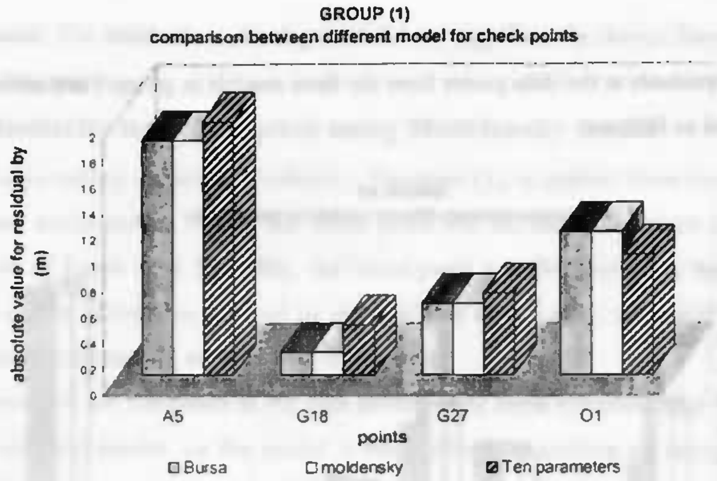


Figure (3) Resultant residuals of group (1) models at check points

From the above table and figure, it can be seen that the similarity transformation (Bursa and Molodensky) give small residuals than the ten parameters model at three points from the four check points. Bursa and Molodensky models give the same results. So, the best model at check points is (similarity transformation), Bursa or Molodensky.

5-2 Group (2); 2D Surface Polynomials

The practical applications in 2-dimensions will be done for all cases and using all possible orders. Two main branches will be treated, homogeneous and non homogeneous cases.

5-2-1 2D Homogeneous Case

a- Homogenous Curvilinear Normal Case

The first, second and third order polynomials are applied as in Equations (5). The total number of unknowns in the third order equations are 9 unknowns. So two more points are required to have redundancy, and these points are (O1), and (A5).

For the different polynomial orders, the residuals with their statistics of φ and λ are computed at the used data points. The resultant residuals are also computed with their statistics. From the results, it is found that the second order is better than the first and the third is better than the second order polynomials at the solution points.

The resultants of the residuals are also computed at the check points. In inverse proportion with the cases of solution points, the polynomial has bigger residuals with higher orders at the check points. This means that high order polynomials fit better the data but they are not modeling nicely the situation.

b- Homogenous Curvilinear Conformal Case

Equations (6) are applied for first, second and third orders in both cases of φ and λ. The residuals at the data points are computed. The resultant residuals of (φ ,λ) are also computed with their statistics.

At the solution points, the third order is better than the other two. The resultant residuals of (φ ,λ) are also computed at the check points. At the check points, the first order is better than the other two.

c- Homogenous Curvilinear - Multiple Regression Polynomial

c-1 Multiple Regression Separated Coordinates

Equations (7) are applied with four, five, six, seven, and eight terms in both cases of φ and λ using the available data points. The residuals at the data points are computed. The resultant residuals are also computed with their statistics. It is found that the eight-term polynomial is the best at the data points.

The resultant residuals of (φ ,λ) at the check points are computed. It is found that the six terms polynomial is better than the others at the check points.

c-2 Homogenous Curvilinear Multiple Regression–Mixed Coordinates

Equations (8) are applied, the residuals at the data points, in both cases of φ and λ are computed with their statistics. It is found that while the terms of the polynomial increase, the values of the residuals decrease and the best polynomial in fitting the solution points is the one of 7 terms.

The resultant residuals of (φ ,λ) at check points are computed. Statistics of the resultant residuals at the check points are computed. The five terms polynomial gave less residuals than the other polynomial at the check points.

d- The best model among 2D Polynomial- Homogenous cases.

Recalling that, the applied polynomials, till now, are the 2D homogenous ones. The comparison will be among the resultant residuals of φ ,λ. To conclude a result, the following statistics are computed at the data points;

Table (4) The statistics of resultant residuals at the data points in m.

Point	Normal	Conformal	Multiple regression	Multiple regression
	3-order	3-order	Separated 8-terms	mixed 7-terms
Min.	0.00	0.01	0.12	0.06
Max	0.01	0.13	0.26	0.66
Mean	0.00	0.08	0.18	0.29
ST.dev.	0.00	0.04	0.05	0.19

At the data (solution) points, the best polynomial is 3rd order normal then 3rd order conformal then 8-terms separated multiple regression then 7-terms mixed multiple-regression. The results of the best solution, at check point, are collected as;

Table (5) The resultant residual values at check points in m.

Point	Normal	Conformal	Multiple regression	Multiple regression
	1-order	1-order	Separated 6-terms	Mixed 5-terms
A5	0.90	0.90	0.87	0.65
G18	0.57	0.57	0.72	1.11
G27	0.54	0.54	0.58	0.25
O1	0.73	0.73	0.33	0.31

The best solution at check points is the fifth order-multiple regression mixed case. This solution is a little bit better than the 6-terms separated multiple regression. The resultant residuals from all above solutions are represented as follows;

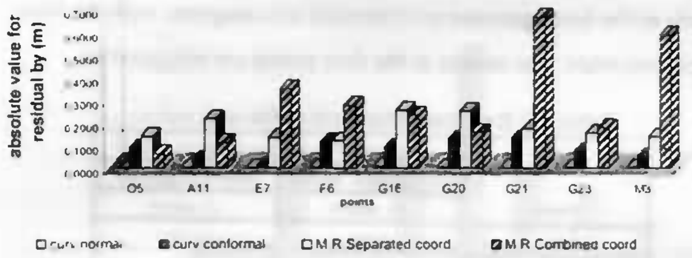


Figure (4) Resultant residuals from best solutions for 2D polynomial homogenous case at data points

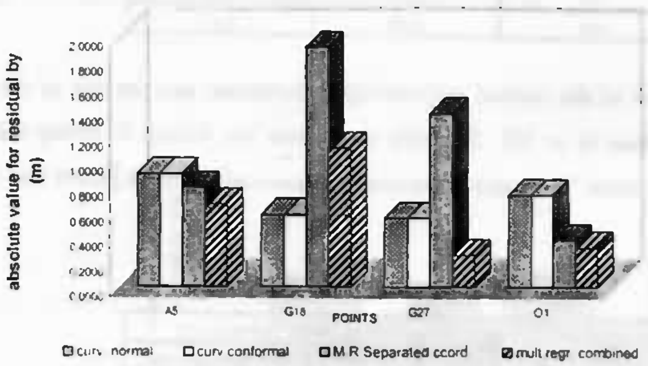


Figure (5) Resultant residuals from best solutions for 2D polynomial homogenous case at check points

5-2-2 Non -Homogenous 2D Polynomials

Non-Homogenous means relating (X,Y, Z) of one system to the corresponding (φ ,λ) of the other system. Equations (9) are applied in three orders and the coefficients are computed. The residuals in X, Y, and Z coordinates, at the data points are also computed with the statistics of their resultants.

It is found that the third order gives less residuals than the other two, at the solution points. The resultant residuals are also computed at the check points. The first order polynomial is better at the check points.

5-3-3 The Best Model in 2D Polynomials Group

The best models in the homogeneous polynomials are compared with the Non-homogenous polynomials. The results at the data points are tabulated as;

Table (6) Resultant residuals at the data points

Points	Resultant for solution points by (m)	
	Homogenous. Normal (3- order)	Non-Homogenous. (3- order)
O5	0.00	0.01
A11	0.01	0.17
E7	0.00	0.01
F6	0.00	0.15
G16	0.01	0.30
G20	0.00	0.12
G21	0.00	0.09
G23	0.00	0.04
M3	0.00	0.01

Although the resultant of the normal polynomial is horizontal and the one of the non-homogenous polynomial is in 3D. the table shows that the former is fitting the data points better than the latter. The same comparison is made at the check points and it was as follows;

Table (7) Comparison at check points

Points	Resultant for check points by (m)	
	Homogenous. Normal (5- terms)	Non-Homogenous (1- order)
A5	0.51	1.67
G18	0.27	0.41
G27	0.53	0.61
O1	0.00	1.00

Again. the homogenous normal is better than the non-homogenous polynomial. The comparison is represented at the solution points as;

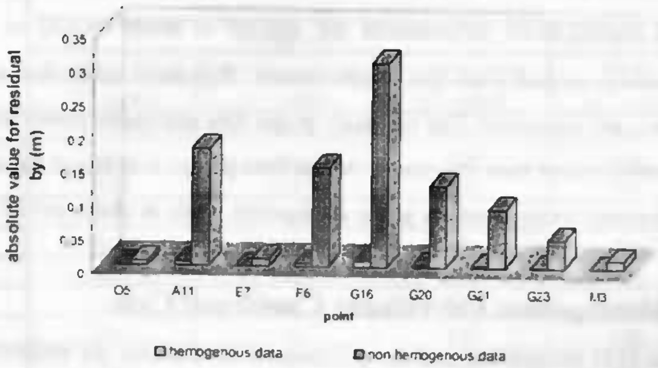


Figure (6) Resultant residuals from the best models for 2D polynomials at data points

At the check points, the comparison is represented as;

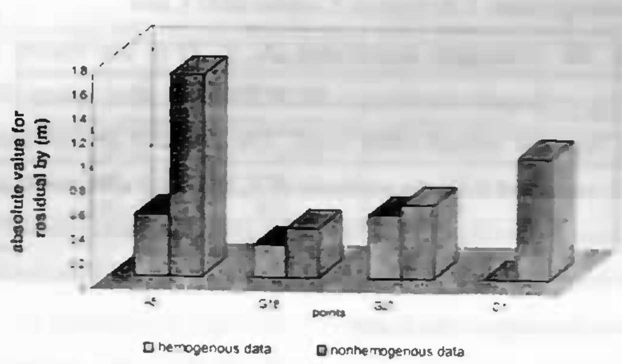


Figure (7) Best models for 2D polynomials at check points

Concerning the 2-D polynomials and as a final conclusion, the best model at data points is 3rd order normal polynomial. The best model at the check points is five terms mixed multiple regression.

5-3 Three Dimension Surface Polynomials

The transformation of coordinates in 3 dimensions using different polynomials is investigated. The results will be discussed and represented in the subsequent sections.

a- 3D Homogenous Curvilinear Normal Polynomial

First and second order polynomials are applied to relate (φ,λ,h) of a datum to the corresponding (φ',λ',h') of the other datum. Equations (10) are applied and the coefficients are computed. The residuals at the data and check points are also computed with the statistics of their resultants. At the data points, it is found that the second order gives minimum residual values at the data points while at check points the first order is better

b- 3D Homogenous Curvilinear Conformal Case

Equations (11) are applied and the polynomial coefficients are obtained. The residuals are computed. The resultant residuals are computed with their statistics. It is found that the second order is better than the first order polynomial at the data points. At the check points, the first order is better than the second order.

c- 3D Homogenous Cartesian Conformal Case

In this case, the rectangular coordinates (X, Y, Z) of one system are related to the corresponding rectangular coordinates (X',Y',Z') of the other system. Equations (12) are applied, in first and second order cases. The residuals are computed. The resultant residuals are computed from the residuals of X,Y,Z at the data points and their statistics. It is found from the results that the first order gives minimum residual values at the data points. The minimum residuals at check points are obtained from the first order.

d- 3D Non-homogenous Case

In this model, the rectangle coordinates (X, Y, Z) in one system are related to the corresponding curvilinear coordinates (φ, λ, h) in the other system. Equations (13) are applied and the residuals of (X, Y, Z) equations are computed at the data points. The resultant residuals and their statistics are computed.

It is found from the results that the second order gives better residual values at the data points while better residuals are obtained from the first order at the check points.

5-3-1 The Best Model Among 3-D Polynomials

The best four previous 3D polynomials are collected here for the sake of comparison and choosing the best model among them. First, the resultant residuals at the data points are tabulated, from the different solutions as follows:

Table (8) Comparison between all models used in 3D polynomial at data points.

Points	Resultant Residuals at solution points by (m)			
	Homogenous			Non Homogenous
	Normal curvilinear (2nd order)	Conformal curvilinear (2nd – order)	Cartesian normal (1st - order)	Normal (2nd - order)
O5	0.03	080	0.52	0.03
A11	0 54	0.65	0.38	0.54
E7	0.29	0.78	0.72	0.29
F6	0.17	1.35	0.76	0.17
G16	0.05	0.75	0.69	0.05
G20	0.54	0.33	0.20	0.53
G21	0.02	0.30	0.53	0.02
G23	0.26	0.39	0.67	0.26
M3	0.07	1.36	1.10	0.07

The results of the 2^{nd} order homogenous normal curvilinear and the 2^{nd} order non-homogenous polynomials are identical. They are better than the other two.

The resultant residuals, at the check points, are also collected as follows:

Table (9) Comparison between all models used in3D polynomial at check points.

Points	Resultant Residuals at check points by (m)			
	Homogenous			Non Homogenous
	Normal curvilinear (1st - order)	Conformal curvilinear (1st – order)	Cartesian normal (1st - order)	(1st - order)
A5	2.02	1.99	1.97	1.88
G18	0.82	0.80	0.75	0.53
G27	1.02	0.85	0.60	0.74
O1	0.90	0.91	0.82	0.90

At check points, the homogenous Cartesian 1^{st} order and the non-homogenous 1^{st} order are the best and they are very close to each other.

The resultant residuals from the best four solutions, at data points are represented as:

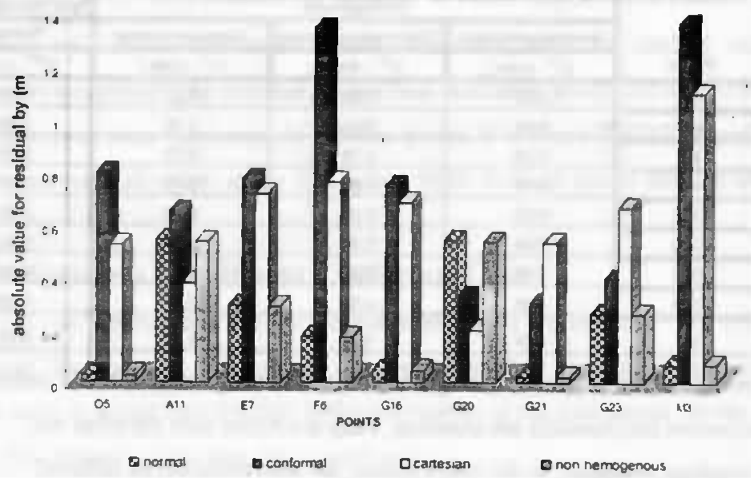


Figure (8) Comparison between best models used in 3D polynomial at data points

The resultant residuals at the check points are represented as;

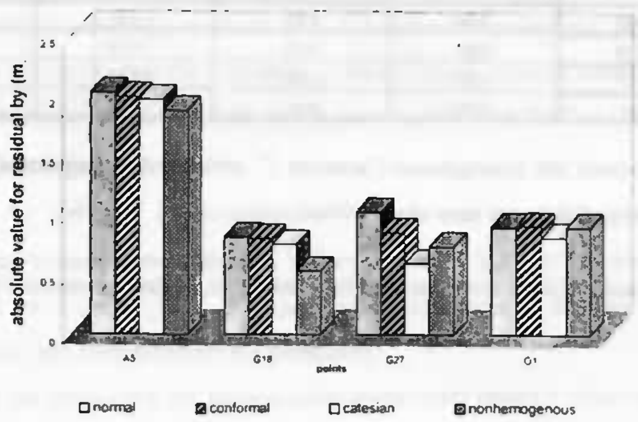


Figure (9) Comparison between best models used in 3D polynomial at check points

5-4-2 The Best Model Among all 3D Models

Recalling that, at data points, the best model of Group1 solutions is chosen as 10 parameters model. Among all (3D) polynomials, the 2nd order non-homogenous or the 2nd order homogenous curvilinear polynomial is chosen as the best solution. The following comparison, between these two solutions is made to choose the best solution at the data points;

Table (10) The comparison between best 3D models at data points.

Points	Resultant residuals at solution points by (m)	
	(G3) polynomial (2^{nd} order)	(G1) Affine transformation
	Non-Homogenous normal	10 – parameters
O5	0.03	0.67
A11	0.54	0.36
E7	0.29	0.78
F6	0.17	0.94
G16	0.05	0.58
G20	0 53	0.22
G21	0.02	0.22
G23	0.26	0.44
M3	0.07	1.82

At the data points. the best 3D solution is the non-homogenous polynomial. The homogenous curvilinear polynomial gives the same best results. At the check points and concerning the 3D models. Bursa and Molodensky were the best models. Among the 3D polynomials. the 1^{st} order homogenous Cartesian and the 1^{st} order non-homogenous polynomials were the best. The comparison between the best two models to chose the best, as follows;

Table (11) The comparison between best models at check points.

Points	Resultant residuals for check points by (m)	
	Group (1) Similarity	Group (3) polynomial (2D)
	Bursa	Non- homogenous 1^{st} order
A5	1.81	1 88
G18	0 18	0 53
G27	0.55	0 74
O1	1.10	0.90

At the check points. Bursa model is the best among all solutions. The following figures represent the data in the above two tables

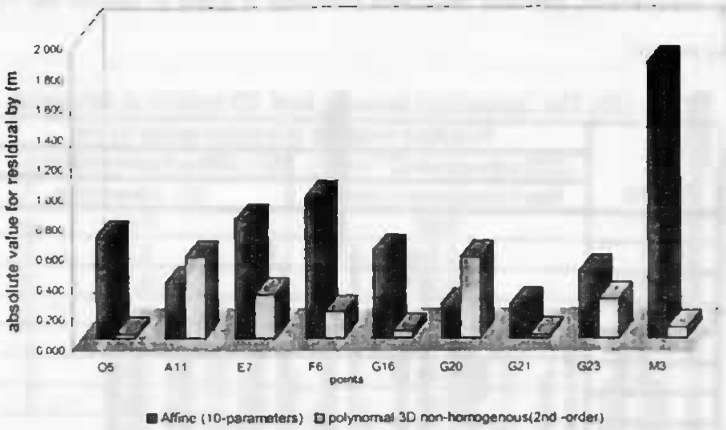


Figure (10) Comparison between best models used in all groups at data points

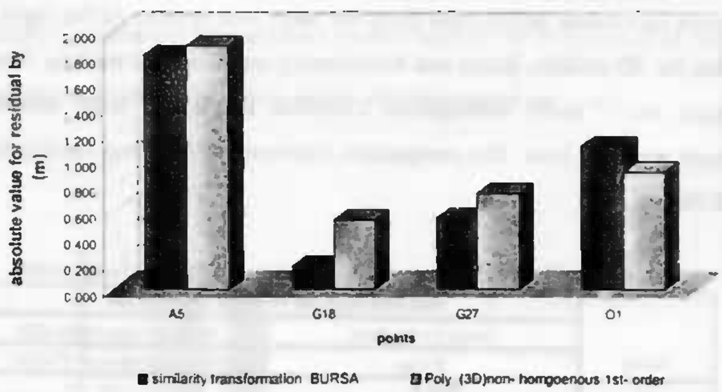


Figure (11) Comparison between best models used in all groups at check points

As a final conclusion concerning the (3D) case, the best model at data points is the 2nd-order non homogenous and the homogenous curvilinear polynomials. The best solution at check points is Bursa or Molodensky model.

Concerning the (2D) solutions, it should be mentioned also that the best model, at data points, is the 3rd order normal polynomial. At the check points, the best model is the five terms mixed multiple regressions.

6- Conclusions

Looking at all results, tabulated and illustrated in histograms, the following could be concluded:

1- Both of Bursa and Molodensky models behave the same towards the data and the check points.

2- The different assumptions of the central (initial) point in Molodensky model do not affect the results of the model. So, real or imaginary point could be used.

3- Changing the central (initial) point in ten-parameters model does not affect the results exactly like in Molodensky model.

4- Ten-parameters model gave slightly better results than Bursa and Molodensky models.

5- Concerning 2D and 3D polynomials, the best model at the data points is not necessary to be the best one at the check points. The former has always higher order than the later.

6- The higher order polynomial curves itself more to fit the data points as possible as it can. So, it gives small residuals at those points.

7- The check points do not share in forming the surface of the polynomial, so it is expected to have large residuals than those of the data points.

8- Generally, the polynomials do not suit the geodetic datum like the similarity and affine transformation models.

9- Polynomials need dense common points to represent the relation between the two concerning systems in good way. Then the residuals at the check points will be improved.

Based on what are concluded, the following can be recommended:

1- Similarity transformation models could be used when few common points are available.

2- Polynomials should be used in transformation when large number of common points are available.

3- Concerning the 2D transformations, the best model at the data points is the 3rd order normal polynomial and the best model at the check points is five terms mixed multiple regression.

4- Concerning the 3D transformations, the best model at the data points is the 2^{nd} order non homogeneous and the homogeneous curvilinear polynomials and the best model at the check points is Bursa and Molodensky models.

5- The behavior of the transformation model towards the data points is not as important for the user as the precision of the model at the check points. The user's points can be considered as check points. Therefore, the best model in 2D transformation is the five terms mixed multiple regression model, while the best model in 3D transformation is Bursa or Molodensky model.

7- References

1. **Abd-Elhay, A. (2004)**: "Investigating the Performance of the Different Coordinate Transformation Models", M.Sc. Thesis, Shoubra Faculty of Engineering, Cairo.
2. **Abd-Elmotaal, H. (1994):** "Comparison of Polynomial and Similarity Transformation Based Datum Shift for Egypt' Bulletin Gèodèsique 1994-68.
3. **Awad, M. E. (1997):** "Studies Towards the Rigorous Adjustment and Analysis of the Egyptian Primary Geodetic Networks Using Personal Computer", Ph.D. Thesis, Faculty of Engineering, Ain-Shams University, Cairo, Egypt
4. **Shaker, A. A. (1982)**: " Three Dimensional Adjustment and Simulation of the Egyptian Network" Ph. D. Thesis, the Institute of the Mathematical and Physical Geodesy, Graz, Austria.
5. **Shaker, A. A. (1983)**: "Geodesy Part II", Lecture notes, Faculty of Engineering-Shoubra, Cairo, Egypt.
6. **Fayad, A. T. (1996)**: "Merging Both GPS and Terrestrial Data in The Computations of the Geodetic Control Networks" Ph. D. Thesis, Faculty of engineering, Ain Shams University, Cairo, Egypt.
7. **Bekheet, A. A. (1993)**: "A Compartive Study Among Different Transformation Models" M.Sc Thesis, Faculty of Engineering- Shoubra, Cairo, Egypt.
8. **El.-Tokhey, M. E. (1999):**" Transformation Parameters Between GPS and the Egyptain Geodetic Reference System" Bulletin of Faculty of Engineering, Ain Shams University, Cairo, Egypt.
9. **Anderson, J. M. and Mikhail, E. M. (1998)**: "Surveying, Theory and Practice", McGraw-Hill Companies, Boston, USA.

10. **ESA Report, (1997): "Introducing the New** Egyptian Datum", ESA, Cairo, Egypt

11. **Finnmap project (1989):** Egyptian Survey Authority. Final Report, Eastern Desert Topographic Mapping Project.

12. **Gomaa, M. and D. S. Alnaggar (2000):** " Optimum Geodetic Datum Transformation Techniques for GPS Surveys in Egypt" Al-Azher Engineering Sixth International Conference. Volume 4.Septemper 2000, Cairo, Egypt.

13. **Heiskanen,W.A. and H. Moritz (1967):** "Physical Geodesy "W.H. Freeman and Co., San Francisco and London 1967.

14. **Nassar, M. M. (1981):** "Matrix Treatment of Adjustment Computations in Surveying", Lecture Notes No 4, Dept of Public Works, Faculty of Engineering, Ain Shams University, Cairo, Egypt.

15. **Saad, A. (1998):** "Unification of the GPS work in Egypt", Civil Engineering Research Magazine, Faculty of Engineering, Al-azhar University, Cairo, Egypt.

16. **Thomson, D.B. (1976):** "Combination of Geodetic Networks", Technical Report No. 30, Surveying Engineering, University of New Brunswick, Fredericton, Canada.